Superconducting Spiral Phase in the two-dimensional t-J model

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We analyse the $t-t'-t''-J$ model, relevant to the superconducting cuprates. By using chiral perturbation theory we have determined the ground state to be a spiral for small doping $\delta \ll 1$ near half filling. We evaluate the spin-wave Green's functions and address the issue of stability of the spiral state, leading to the phase diagram of the model. At $t' = t'' = 0$ the spiral state is unstable towards a local enhancement of the spiral pitch, and the nature of the true ground state remains unclear. However, for values of t' and t'' corresponding to real cuprates the (1,0) spiral state is stabilized by quantum fluctuations ("order from disorder" effect). We show that at $\delta \approx 0.119$ the spiral is commensurate with the lattice with a period of 8 lattice spacings. It is also demonstrated that spin-wave mediated superconductivity develops in the spiral state. Even though one cannot classify the gap symmetry according to the lattice representations (s,p,d,...) since the symmetry of the lattice is spontaneously broken by the spiral, the gap always has lines of nodes along the $(1, \pm 1)$ directions.

I. INTRODUCTION

The $t-J$ model has been suggested to describe the essential low-energy physics of the high-T_c cuprates [1–3]. Formation of spirals in the doped $t-J$ model was proposed by Shraiman and Siggia [4]. They showed that the pitch of the spiral is proportional to the hole concentration δ, see also Ref. [5]. The idea that the spiral state is the ground state of the doped Heisenberg antiferromagnet attracted the attention of theorists [6–13], however the question of stability of the state remained controversial. According to Refs. [6, 8, 10] the spiral state is unstable toward a local enhancement of the spiral pitch. On the other hand the analysis of Ref. [9] indicated that the spiral state is stable. A semiclassical analysis of stability of the (1,0) spiral state was performed in Ref. [5]. According to this analysis the state is marginal (zero stiffness), which effectively indicates that the state is unstable. A complete stability analysis of spiral states in the Hubbard model was performed in Ref. [11] also using the semiclassical approximation. According to their analysis in the leading in powers of doping approximation the (1,1) spiral is always unstable and the (1,0) spiral is always marginal in agreement with [5]. Recently the interest in the spiral state was renewed because of the strong experimental indications that at small doping the cuprates behave as spin glasses or even exhibit some kind of magnetic ordering. A summary of the available experimental data on one of the superconducting cuprates, $La_{2-\delta}Sr_{\delta}CuO_4$, is given in Ref. [14]. The data also show that magnetic ordering and superconductivity coexist. The spin glass behavior is consistent with the spiral scenario: since doping is not uniform the pitch of the spiral is varying from point to point and hence on large scales it leads to spin glass behavior [15].

In the present work we analyse the stability of spiral states within the RPA approximation. The approximation is parametrically justified for $\delta \ll 1$. In this part of the work on the technical side we follow the approach developed by Igarashi and Fulde [9]. However, contrary to them and in agreement with Refs. [6, 8], we conclude that in the "pure" $t-J$ model ($t' = t'' = 0$) the spiral state is unstable with respect to a local enhancement of the spiral pitch. We find that relatively small values of t' and t'' stabilize the spiral order, with a uniform hole distribution. Our results are then consistent with numerical (DMRG) results indicating that inhomogeneous (striped) phases disappear with the increase of further-neighbor hoppings [16, 17]. In our approach the instability of the spiral is closely related to the fact that the hole dispersion is almost degenerate along the face of the magnetic Brillouin zone. As soon as the degeneracy is sufficiently lifted by t' and t'' the instability disappears, leading to a stable (1,0) spiral state. Within the semiclassical approximation the effective stiffness of this state is zero in agreement with [5, 11]. However spin quantum fluctuations give rise to a nonzero positive stiffness and hence favor stability via an "order from disorder" mechanism. For parameter values corresponding to real cuprates, the pitch of the spiral is proportional to doping, and at $\delta \approx 0.119$ the spiral becomes commensurate with the lattice with period 8, in agreement with the experimental data of Tranquada *et al* [18, 19].

The possibility of s- and d-wave pairing between mobile holes due to a static distortion of the Neel background was pointed out in Ref. [20]. Such a distortion leads to an infinite set of very shallow two-hole bound states [21]. In Ref. [22] it was shown, under the assumption that Neel order is preserved under doping, that in the $t-J$ model there is spin-wave mediated superconducting pairing in all waves except the s-wave. The pairing was found to be maximum in the d-wave channel. A numerical calculation performed in Ref.[23] under the same assumption showed that the pairing was quite strong. However, the picture of pairing [22, 23] was not consistent because the starting point of the analysis was the (unstable) Neel background. In the present paper, after proving the stability of the spiral state, we consider superconducting pairing on this state. Using an approach similar to that of Ref.[22] we find a superconducting pairing instability and show that the gap always has lines of nodes along $(1, \pm 1)$ directions.

II. HAMILTONIAN

The Hamiltonian of the $t - t' - t'' - J$ model is:

$$H = -t \sum_{\langle ij \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} - t' \sum_{\langle ij_1 \rangle \sigma} c^{\dagger}_{i\sigma} c_{j_1\sigma} - t'' \sum_{\langle ij_2 \rangle \sigma} c^{\dagger}_{i\sigma} c_{j_2\sigma} + J \sum_{\langle ij \rangle \sigma} \left(\mathbf{S}_i \mathbf{S}_j - \frac{1}{4} n_i n_j \right) . \tag{1}$$

$c^{\dagger}_{i\sigma}$ is the creation operator of an electron with spin σ ($\sigma =\uparrow, \downarrow$) at site i of the two-dimensional square lattice, $\langle ij \rangle$ represents nearest neighbor sites, $\langle ij_1 \rangle$ - next nearest neighbors (diagonal), and $\langle ij_2 \rangle$ represents next next nearest sites. The spin operator is $\mathbf{S}_i = \frac{1}{2} c^{\dagger}_{i\alpha} \sigma_{\alpha\beta} c_{i\beta}$, and the number density operator is $n_i = \sum_{\sigma} c^{\dagger}_{i\sigma} c_{i\sigma}$. The size of the exchange measured in two magnon Raman scattering [24, 25] is $J = 125$meV. Calculations of the hopping matrix elements have been performed by Andersen *et al* [26]. They consider a two-plane situation and the effective matrix elements are slightly different for symmetric and antisymmetric combinations of orbitals between planes. After averaging over these combinations we obtain: $t = 386$meV, $t' = -105$meV, $t'' = 86$meV. From now on we set $J = 1$. In these units we have:

$$t = 3.1, \quad t' = -0.8, \quad t'' = 0.7 \tag{2}$$

An analysis of angle-resolved-photoemission spectra for the insulating copper oxide $Sr_2CuO_2Cl_2$ performed in Ref. [27] with the Hamiltonian (1),(2) shows an excellent agreement with experiment for both the single-hole dispersion and for the photoemission intensity.

III. RESULTS AND CONCLUSIONS

We have studied the phase diagram of the $t - t' - t'' - J$ model close to half-filling. We show that the Neel state is unstable with respect to decay to spiral states as soon as doping is introduced. A picture of the spiral state is shown schematically in Fig. 1 We find that at $t' = t'' = 0$ the spiral state is unstable toward a local enhancement of the

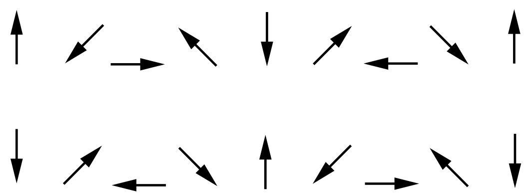
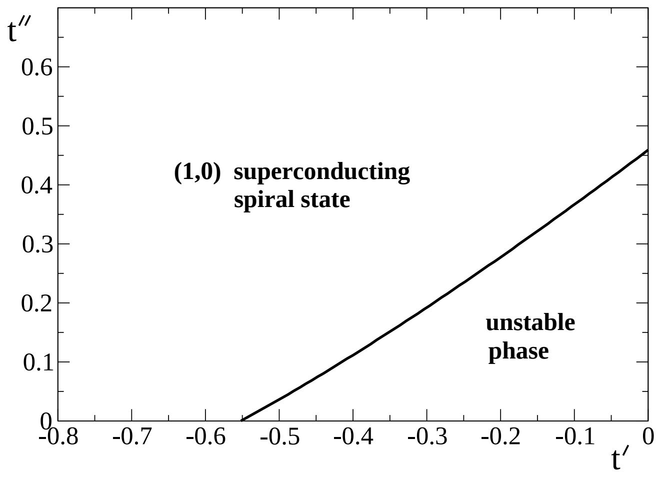


FIG. 1: *(1,0) Spiral state for* $\delta = 0.119$*, corresponding to* $Q = \pi/4$*.*

spiral pitch, and consequently at that point the nature of the true ground state remains unclear. However we have shown that for values of t' and t'' corresponding to real cuprates the (1,0) spiral state is stable. The phase diagram of the model at $t = 3.1$ (we set $J = 1$) is shown in Fig.2 left. For hole concentration $\delta \approx 0.119$ the (1,0) spiral is commensurate with the lattice with a period of 8 lattice spacings, in agreement with experimental data. Fig.2 right shows the dependence of the spiral on-site magnetization on doping.

We demonstrate that spin-wave mediated superconductivity is developed above the spiral state and derive analytically a lower limit for the superconducting gap. One cannot classify the gap according to the lattice representations A_1, A_2, B_1, B_2, and E ("s","d","p",...) since the symmetry of the lattice is spontaneously broken by the spiral. However, the gap always has lines of nodes and a symmetry similar to d-wave.

It is likely that some kind of order in the charge sector appears around the point $t' = t'' = 0$, where the spiral state is inherently unstable. If we indeed interpret the region marked as "unstable" on our phase diagram Fig.2 left as a candidate for such order, then the effect of increasing t' and/or t'' is to drive the system towards the (stable) spiral order, with a homogeneous charge distribution. This occurs for $|t'|/t \gtrsim 0.18$ (at $t'' = 0$). From this point of view our results are similar to the DMRG results describing the destruction of stiped phases by second-neighbor hopping



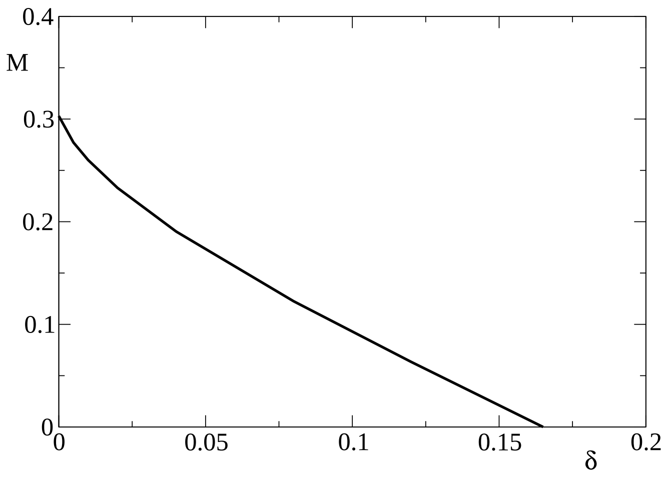


FIG. 2: *The left figure shows phase diagram of the $t - t' - t'' - J$ model at $t = 3.1$ and small doping, $\delta \ll 1$. The bottom right corner corresponds to the pure $t - J$ model, $t' = t'' = 0$. The top left corner corresponds to parameters from Ref.[26], Eq. (2). The region of stability of the superconducting (1,0) spiral phase is shown.*
The right figure shows on-site magnetization in the (1,0) spiral versus doping for $t = 3.1$, $t' = -0.8$, and $t'' = 0.7$.

[16, 17], where a critical value of t' necessary to destabilize the stripes was found, and is quite close to our estimate above.

[1] P. W. Anderson, Science **235**, 1196 (1987).
[2] V. J. Emery, Phys. Rev. Lett. **58**, 2794 (1987).
[3] F. C. Zhang and T. M. Rice, Phys. Rev. B **37**, 3759 (1988).
[4] B. I. Shraiman and E. D. Siggia, Phys. Rev. Lett. **62**, 1564 (1989).
[5] B. I. Shraiman and E. D. Siggia, Phys. Rev. B **46**, 8305 (1992).
[6] T. Dombre, J. Phys. (France) **51**, 847 (1990).
[7] C. L. Kane, P. A. Lee, T. K. Ng, B. Chakraborty, and N. Read, Phys. Rev. B **41**, 2653 (1990).
[8] A. Auerbach and B. E. Larson, Phys. Rev. B **43**, 7800 (1991).
[9] J. Igarashi and P. Fulde, Phys. Rev. B **45**, 10419 (1992).
[10] H. Mori and M. Hamada, Phys. Rev. B **48**, 6242 (1993).
[11] A. V. Chubukov and K. A. Musaelian, Rev. B **51**, 12605 (1995).
[12] B. Normand and P. A. Lee, Phys. Rev. B **51**, 15519 (1995).
[13] L. O. Manuel and H. A. Ceccatto, Phys. Rev. B **61**, 3470 (2000).
[14] M. -H. Julien, Physica B **329-333**, 693 (2003).
[15] N. Hasselmann, A. H. Castro Neto, and C. Morais Smith, cond-mat/0306721.
[16] S. R. White and D. J. Scalapino, Phys. Rev. B **60**, R753 (1999).
[17] T. Tohyama, C. Gazza, C. T. Shih, Y. C. Chen, T. K. Lee, S. Maekawa, and E. Dagotto, Phys. Rev. B **59**, R11649 (1999).
[18] J. M. Tranquada, B. J. Sternlieb, J. D. Axe, Y. Nakamura, and S. Uchida, Nature **375**, 561 (1995).
[19] J. M. Tranquada, J. D. Axe, N. Ichikawa, Y. Nakamura, S. Uchida, and B. Nachumi, Phys. Rev. B **54**, 7489 (1996).
[20] B. I. Shraiman and E. D. Siggia, Phys. Rev. B **40**, 9162 (1989).
[21] M. Yu. Kuchiev and O. P. Sushkov, Physica C **218**, 197 (1993).
[22] V. V. Flambaum, M. Yu. Kuchiev, and O. P. Sushkov, Physica C **227**, 267 (1994).
[23] V. I. Belinicher, A. L. Chernyshev, A. V. Dotsenko, and O. P. Sushkov, Phys. Rev. B **51**, 6076 (1995).
[24] Y. Tokura, S. Koshihara, T. Arima, H. Takagi, S. Ishibashi, T. Ido, and S. Uchida, Phys. Rev. B **41**, 11657 (1990).
[25] M. Greven, R. J. Birgeneau, Y. Endoh, M. A. Kastner, B. Keimer, M. Matsuda, G. Shirane, and T. R. Thurston, Phys. Rev. Lett. **72**, 1096 (1994).
[26] O. K. Andersen, A. I. Liechtenstein, O. Jepsen, and F. Paulsen, J. Phys. Chem. Solids **56**, 1573 (1995).
[27] O. P. Sushkov, G. A. Sawatzky, R. Eder, and H. Eskes, Phys. Rev. B **56**, 11769 (1997).