Quantum Interference with Heisenberg Spin Chains

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It is now becoming possible to arrange spins, for example on quantum dots, in arbitrary configurations and to control the Heisenberg interaction between each pair of spins. As a result, traditional quantum interference experiments can be performed in new ways. An application to interference in conjunction with spin-exchange is proposed.

1. Introduction

The exchange interaction between electron spins on arrays of atoms or quantum dots is one of the physical interactions that could allow the construction of a quantum computer [1]. This is possible for two main reasons. First, the quantum dots on which the spins are localised can be arranged in interesting configurations and, second, the exchange interaction between any pair of spins can be varied in strength or turned on and off at will [2]. Another consequence of these developments is that chains of spins could be used to implement quantum interference experiments such as a photon in a Mach-Zehnder interferometer or an electron in a two-slit experiment. The purpose of the present paper is to investigate this possibility and to see whether it offers advantages over more conventional approaches to quantum interference phenomena.

2. Theory

The exchange interaction between the pairs of spins on quantum dots can be made to be isotropic [3]. The time-dependent exchange coupling $J_{mn}(t)$ between the nearest-neighbour spins m and n with spin operators $\mathbf{S}_m$ and $\mathbf{S}_n$ is then described by the Hamiltonian

$$H_{mn}(t) = J_{mn}(t)\mathbf{S}_m \cdot \mathbf{S}_n \,. \tag{1}$$

With this Hamiltonian, the solution of the time-dependent Schrödinger equation is

$$\psi(t) = \exp\left(-\frac{i}{\hbar}\int_0^t J_{mn}(t')\,dt'\,\mathbf{S}_m \cdot \mathbf{S}_n\right)\psi(0). \tag{2}$$

Eq. (2) can be written in the dimensionless form [4]

$$\psi(\tau) = \exp(-i2\pi\,\tau_{mn}\,\mathbf{S}_m \cdot \mathbf{S}_n)\,\psi(0) \quad \text{where} \quad \tau_{mn} = \frac{1}{2\pi\hbar}\int_0^t J_{mn}(t')\,dt' \tag{3}$$

and $\psi(\tau)$ is the solution of the dimensionless equation

$$i\frac{\partial\psi(\tau)}{\partial\tau} = V_{mn}\psi(\tau) \quad \text{where} \quad V_{mn} = 2\pi\mathbf{S}_m \cdot \mathbf{S}_n \,. \tag{4}$$

We will consider an arrangement of 16 spins in which one spin has $S_z = -1/2$ (spin down) in units of $\hbar$ and the rest have $S_z = 1/2$ (spin up). The interaction in Eq. (1) preserves the total $S_z = (N-1)/2$ of the state and a complete set of basis states are $\left|j\right\rangle, j = 1,16$ where $\left|j\right\rangle = \left|0_1 0_2 \ldots 1_j \ldots 0_{N-1} 0_N\right\rangle$ and $0_i\,(1_i)$ represents spin up (down) at the i^{th} site.

The effect on the basis states of pulsing the interaction $J_{nm}(t)$ for a time t can be obtained from Eqs. (3) and (4):

$$V_{mn}(\tau)|m\rangle = e^{i\pi\tau/2}\big(\cos\alpha|m\rangle - i\sin\alpha|n\rangle\big)$$
$$V_{mn}(\tau)|n\rangle = e^{i\pi\tau/2}\big(-i\sin\alpha|m\rangle + \cos\alpha|n\rangle\big) \tag{5}$$
$$V_{mn}(\tau)|j\rangle = e^{-i\pi\tau/2}|j\rangle,\ j \neq m,\ j \neq n$$

where $V_{mn}(\tau)$ indicates that the interaction is non-zero over the interval 0 to τ, $\alpha = \pi\tau$ and τ is given in Eq. (3). We see that the effect of $V_{mn}(1/2) \equiv U_{mn} (= U_{nm})$ is to swap the spins on the m^{th} and the n^{th} sites and change the phase by $-i\pi/4$: $U_{mn}|\ldots 0_m 0_n \ldots\rangle = e^{-i\pi/4}|\ldots 0_m 0_n \ldots\rangle$, $U_{mn}|\ldots 0_m 1_n \ldots\rangle = e^{-i\pi/4}|\ldots 1_m 0_n \ldots\rangle$ and $U_{mn}|\ldots 1_m 0_n \ldots\rangle = e^{-i\pi/4}|\ldots 0_m 1_n \ldots\rangle$.

3. Results

The down spin on site 1 in the array of spins shown in Fig. 1 can be made to behave in the same way as a particle in a Mach-Zehnder interferometer by the application of a suitable sequence of pulsed interactions. Firstly the down spin is moved to site 3 by the application of $U_{3,2}U_{2,1}$ (reading from right to left). Then passage through the equivalent of the first beam splitter is produced by $U_{10,3}V_{4,3}(\tau_1)$, so that

$$|1\rangle \rightarrow e^{i(2\alpha-3\pi)/4}(\cos\alpha\,|10\rangle - i\sin\alpha\,|4\rangle) \tag{6}$$

where $\alpha = \pi\tau_1$. Passage through the equivalent of the second beam splitter is produced by $U_{9,8}U_{8,7}U_{7,12}U_{14,13}U_{13,7}V_{12,7}(\tau_2)U_{12,11}U_{11,10}U_{7,6}U_{6,5}U_{5,4}$. After the interaction $V_{12,7}(\tau_2)$ in this sequence, the state is (with $\beta = \pi\tau_2$)

$$e^{i(\alpha+\beta)/2}[\cos\alpha(\cos\beta\,|12\rangle - i\sin\beta\,|7\rangle) - i\sin\alpha\,(\cos\beta\,|7\rangle - i\sin\beta\,|12\rangle)]. \tag{7}$$

The complete transformation at the end of the above sequences is

$$|1\rangle \rightarrow e^{i[2(\alpha+\beta)-5\pi]/4}(\cos(\alpha+\beta)\,|9\rangle - i\sin(\alpha+\beta)\,|14\rangle). \tag{8}$$

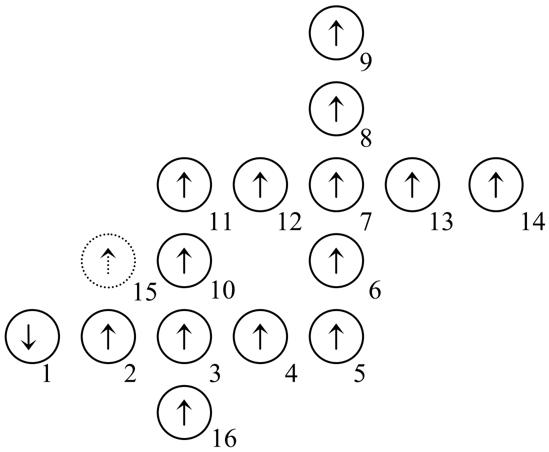

Fig. 1. Array of spins to simulate a Mach-Zehnder interferometer. The numbering of the spins used in the text is shown. The state shown is |1>. With respect to sub-section 3.1, spin 15 belongs to Bob, the rest to Alice.

Thus just as in a conventional Mach-Zehnder interferometer, the down spin can be made to appear in one or other of the output arms, i.e. at |9> or |14> or a combination of both, by choosing α and β which specify the interactions which act as the equivalents of beam splitters.

The implementation of quantum interference via controlled spin exchange leads to the possibility of new types of experiments. In the next section, we can consider a type of interaction-free measurement [5] with spins.

3.1 Detection of the exchange of "identical" quantum objects

Bob has a quantum dot *m* with spin up and says that with probability of 1/2 he will apply the swap interaction U_{mn} with the spin on a quantum dot n specified by Alice whenever she instructs him to do so. Bob challenges Alice to identify *without error* at least some of the cases in which (i) Bob has carried out the swap and (ii) is left with spin up on his quantum dot *m*. Is it possible for Alice to choose such a subset *without error*, i.e., without including any cases in which (i) Bob did not carry out the exchange or (ii) ended up with a spin which was not down? The answer would appear to be "no" because under Bob's criterion, Bob is left with an object identical with the one he started with (except for a change in phase of the product state of the quantum dots). In fact the answer is "yes", because Alice could use the spin array described in the previous section in the following way.

Alice chooses $\alpha \neq 0, \pi/2, \pi, 3\pi/2 \ldots$ and $\alpha+\beta=2\pi$ so that her spin specified by the state |1> will evolve to dot 9 under the interaction specified in Eqs. (6) to (8). Let Bob's spin be on dot 15 in Fig. 1 and note that his spin up is specified by the initial state |1>. Alice instructs Bob to exchange with her dot 10 in Fig. 1 when she has evolved her state part-way through the interferometer to the state described in Eq. (6). If Bob declines to exchange, which he does in 1/2 the cases, the sequence is that specified in Eqs. (6) to (8) and since we have chosen $\alpha+\beta=2\pi$ in Eq. (8)

$$|1>\rightarrow e^{-i5\pi/4}\,|9>. \qquad (9)$$

If Bob does exchange, which he does in 1/2 the cases, the sequence is

$$\begin{aligned}|1>&\rightarrow e^{i(2\alpha-3\pi)/4}(\cos\alpha|10>-i\sin\alpha\,|4>)\rightarrow e^{i(\alpha/2-\pi)}(\cos\alpha|15>-i\sin\alpha\,|4>)\\ &\rightarrow e^{i(\alpha+\beta-\pi/2)/2}(-i\sin\alpha\,(\cos\beta|\,7>-i\sin\beta\,|12>)-\cos\alpha\,|15>) \qquad (10)\\ &\rightarrow e^{i(\alpha+\beta-3\pi)/2}(-i\sin\alpha\,(\cos\beta|14>-i\sin\beta\,|9>)-\cos\alpha\,|15>).\end{aligned}$$

Alice measures the spin direction on sites 9 and 14. If both are spin up, Alice knows the state is neither |9> nor |14> and she concludes Bob exchanged spins and that his spin is down (i.e., the final state is |15> in Eq. (10)). If Alice finds a spin down on site 9, Alice cannot be sure whether an exchange took place or not because Eq. (9) (no exchange) and Eq. (10) (exchange) both can lead to |9>. Alice finds a spin down on site 14, i.e. the final state is |14>, she can conclude that Eq. (10) applies and therefore that a swap has taken place and that Bob is left with spin up. Bob's challenge has been met.

For practical implementation, the same conclusions can be drawn using far fewer spins but the resemblance to traditional interference experiments is not then so transparent.

4. Conclusion

It has been shown that the isotropic Heisenberg interaction between spins on quantum dots can be used to manipulate spins to demonstrate traditional quantum interference phenomena. The new approach has been applied to a variation on the idea of interaction-free measurement involving the exchange of spins on identical quantum objects.

References

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