The Distinguishability of Quantum States in Self-Assembled Quantum Dots

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A quantum system composed of a pair of coupled quantum dots is examined to find out whether one state can be "collapsed" onto another by a suitable transformation. It is found that although the "collapse" can be partially achieved, the states involved remain orthogonal and therefore potentially very interesting effects would not be able to be demonstrated experimentally.

1. Introduction

A quantum system described in a 2 dimensional (2D) Hilbert space (spanned by two orthogonal states $|a>$ and $|b>$) can be shown physically to occupy a 1D subspace $|c>$ by a unitary transformation like that provided, for example, by a beam-splitter:

$$\alpha_1 |a> + \alpha_2 |b> \to |c> \tag{1}$$

where $\alpha_1^2 + \alpha_2^2 = 1$. If the first quantum state is entangled with a second quantum system (for example, one described by a 2D Hilbert space spanned by orthogonal basis states $|u>$ and $|v>$), under normal circumstances, neither quantum system can be made to occupy a 1D subspace by a unitary transformation, for example

$$\psi = \gamma_1 |a>|u> + \gamma_2 |b>|v> \to |d> (\lambda_1 |u> + \lambda_2 |v>) \tag{2}$$

(with $\gamma_1^2 + \gamma_2^2 = \lambda_1^2 + \lambda_2^2 = 1$) *cannot* occur because the last step requires each of 2 orthogonal states [not a superposition of 2 orthogonal states as in Eq. (1)] to be transformed into a common state. Nevertheless, Zou, Wang and Mandel [1] showed experimentally that it is possible to achieve the second transformation for one of the photons of a biphoton produced by parametric down-conversion. This is achieved by beginning with a state $|\psi>= \gamma_1 |x> + \gamma_2 |y>$ and firstly causing the down-conversion of the state $|x> \to |a>|u>$ and then of the state $|y> \to |b>|v>$ while simultaneously arranging for state $|a>$ to be present in the non-linear crystal while the second down-conversion takes place so that $|a>$ merges into $|b>$ to yield Eq. (2) with $|d>=|b>$. Hence a condensed matter system, namely the down-conversion crystals, have been used to achieve the "collapse" of one state onto another subject to the limitations that the transformation of the first state occurs at the time of creation of the second.

The question arises as to whether other condensed matter systems offer the same opportunity and if so whether there are any limitations like those that apply in [1]. A positive answer would have very interesting and far reaching consequences [2]. Since quantum dots (QD's) can be manipulated in increasingly sophisticated ways, they constitute an ideal system for investigating the question. The answer involves states which are analogous to the so-called super-radiant and subradiant states observed in other 2 level systems [3-5].

2. The quantum dot system considered

Semiconductor self-assembled QD's allow optically generated excitons to be dissociated with the separated electrons and holes stored in coupled QD pairs for ultralong storage times of up to several seconds [6]. If the QD pairs are embedded in the intrinsic region of a *p-i-n* junction (with the hole(electron) dot on the *p*(*n*) side), biasing of the junction appropriately can lead to tunneling from the QD pairs to the hole and electron reservoirs, thereby emptying the QD pairs and producing a current pulse in leads connected to the junction. If we consider two QD pairs in the junction, labeled *u* and *l*, coupled to the electron and hole reservoirs, both of the states $|eh>_u|00>_l$ and $|00>_u|eh>_l$, which are the two ways one exciton can exist in the system of the two QD pairs, can be made to decay to the same state $|00>_u|00>_l|eh>_r$, where $|eh>_r$ represents one extra particle in each of the electron and hole reservoirs, and consequently a current in the leads to the junction. Thus we have a candidate for the collapse of two previously orthogonal states onto a common state as in [2].

We can entangle the QD system with another quantum system by exciting the exciton by a superposition of photon states. Consider a Mach-Zehnder interferometer (MZ) with an initial 50:50 beam splitter (BS) and with one of the QD pairs in each of the middle arms of the MZ (labeled *u* in the upper arm and *l* in the lower). If the MZ is fed with a photon pair $|1,1>$, the first beam splitter yields [7] the path entangled state $(|2,0>+|0,2>)/\sqrt{2}$ ($|n,m>$ will mean the occupancy of the upper(lower) arm is *n*(*m*) at any particular stage in the MZ). Assuming a single exciton is produced by the two-photon state with probability unity, the product state of the photon pair, QD's and the e-h reservoirs evolve as follows

$$|1,1>|00>_u|00>_l|00>_r \rightarrow (|2,0>+|0,2>)\,|00>_u|00>_l|00>_r\,/\sqrt{2}$$
$$\rightarrow (|1,0>|eh>_u|00>_l + |0,1>|00>_u|eh>_l)\,|00>_r\,/\sqrt{2}$$
$$\rightarrow [(t\,|1,0>+r\,|0,1>)\,|eh>_u|00>_l + (-r^*\,|1,0>+t^*\,|0,1>)\,|00>_u|eh>_l]\,|00>_r\,/\sqrt{2} \qquad (3)$$

where the four states are, respectively, before the first BS, after the first BS but before the encounter with the QD pairs, after the encounter with the QD pairs and after the second BS and *t* and *r* are determined by the path lengths in the MZ and by the second BS and $|t|^2+|r|^2=1$. The state $|eh>_u$ describes an exciton excited by one of the photons on the upper path with the *e* and *h* separated in the coupled QD pair on the upper path. Similarly on the lower path. We see that the photon which has not been lost in exciting the exciton emerges with equal probability on the upper and lower output paths of the MZ. However *if* we can straightforwardly empty into the carrier reservoirs the *e* and *h* of the coupled QD pair which is storing the exciton, the final state would evolve to

$$((t+r)\,|1,0>+(-r^*+t^*)\,|0,1>)\,|00>_u|00>_l|eh>_r\,/\sqrt{2} \qquad (4)$$

With this form of the wavefunction, which happens because information about which coupled QD pair stored the exciton is lost, interference between the paths in the MZ can take place and by adjusting the MZ path lengths (and/or the second BS) the photon can be manipulated, for example to emerge on the upper path only by making $r=t$ in Eq. (4). We must now examine the process of "emptying" a QD by tunneling into a carrier reservoir. We shall see that although the information about which QD pair stores the exciton can, indeed, be lost, the desired result of Eq. (4) cannot be achieved.

3. The Results

An exactly solvable model of the tunneling dynamics of a two level quantum system coupled to a macroscopic environment has been applied to the states of QD's [8]. That model can be used to deal with the electron which is in one of the QD's in either the upper or lower coupled QD pairs being considered here. The results for the QD's holding the hole are analogous. It is instructive to deal first with the 2 (assumed identical, with energy ε) QD's coupled to a single discrete level (labeled 1 with energy with energy ε_1) and then consider what happens when that level is the continuum representing the electron reservoir. The Hamiltonian is

$$H=\varepsilon(d_u^\dagger d_u + d_l^\dagger d_l) + \varepsilon_1 d_1^\dagger d_1 + V(d_u^\dagger d_1 + d_1^\dagger d_u + d_l^\dagger d_1 + d_1^\dagger d_l) \qquad (5)$$

where $d_u^\dagger(d_l^\dagger)$ creates an electron on the dot on the upper(lower) path, $|e>_u \equiv |u>$ ($|e>_l \equiv |l>$), etc and the coupling *V* can be taken to be real without loss of generality. The eigenstates of *H* are the antisymmetric state $|->=(|u>-|l>)/\sqrt{2}$ and the states $|x>=\alpha_1\,|+>+\alpha_2\,|1>$,

$|y> = \alpha_2 |+> - \alpha_1 |1>$, where the symmetric state $|+> = (|u> + |l>)/\sqrt{2}$, $\alpha_1 = \sqrt{(1+\beta)/2}$, $\alpha_2 = \sqrt{(1-\beta)/2}$ and $\beta = 1\big/\sqrt{1 + 8V^2/(\varepsilon - \varepsilon_1)^2}$. The corresponding energies are $E_- = \varepsilon$, $E_x = (\varepsilon + \varepsilon_1)/2 + \omega$ and $E_y = (\varepsilon + \varepsilon_1)/2 - \omega$ where $\omega = (\varepsilon - \varepsilon_1)\beta/2$. If an electron is in the upper dot at *t=0*, then at time *t* it is in the state

$$|e,t>_u = \frac{1}{\sqrt{2}} |-> e^{-iE_- t} + \frac{1}{2\sqrt{2}} (\alpha_1 |x> e^{-iE_x t} + \alpha_2 |y> e^{-iE_y t}) \qquad (6)$$

and there is a similar expression for the lower state $|e,t>_l$. Thus the QD electron states in the superposition in the second part of Eq. (3) include Rabi oscillations between $|+>$ and $|1>$ but the $|e,t>_u$ and $|e,t>_l$ states remain at all times orthogonal. Thus the manipulation of the state of the photon emerging from the MZ is not enabled by the coupling to the state $|1>$.

We can now consider what happens if the QD's are coupled to a continuum of states by replacing the third term in Eq. (5) by the Hamiltonian $H_c = \sum_{i=u,l} \sum_k \varepsilon_{ki} c_{ki}^\dagger c_{ki} + U_{ki}(c_{ki}^\dagger d_i + d_i^\dagger c_{ki})$ where the sum on *k* is over the continuum adjoining dot *i=u* or *l*, $c_{ki}^\dagger$ creates an electron in the *k*-th state of the continuum adjoining dot *i*, ε_{ki} is the energy of the *k*-th state of the continuum adjoining dot *i* and U_{ki} are assumed real. In the broad band limit of the continua, the behaviour is governed by relaxation frequencies [8] $\Gamma_i = \sum_k U_{ki}^2 \delta(E - \varepsilon_{ki})$. We assume that $\Gamma_1 = \Gamma_2 = \Gamma$ so that the source of the electron in the continuum state is not identifiable and therefore the continuum states occupied by the electron sourced from either dot are not orthogonal. Depending on the relative magnitude of Γ and *U*, the system may continue to exhibit (damped) Rabi oscillations [8] but whether it does or not, the probability of the electron occupying the QD states which are *symmetric* in |*u*> and |*l*> reduce by a factor $e^{-\Gamma t}$ and they eventually becomes the continuum state. However the *antisymmetric* state $|->$ in Eq. (6) does not decay to the continuum and consequently $|e>_u$ and $|e>_l$ remain orthogonal despite the decay of the symmetric state to the continuum. A similar argument applies to the hole state in Eq. (3).

4. Conclusion

The coupled QD's can undergo only a partial transition onto the continuum of states in the carrier reservoir. Therefore the answer to the question posed in the Introduction is that it is not possible to collapse the two QD states onto the continuum of states to factor out one of the entangled states. The crucial point is that the *e-h* pair in the QD system must be viewed as a combination of symmetric and antisymmetric states. Only the symmetric state is coupled to the continuum and even if it decays into a continuum state in which the electron (hole) from the upper and lower dots are indistinguishable, the antisymmetric state is left unaffected with the result that the manipulation of the state of the emerging photon continues to be precluded.

References

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